Three wave mixing of airy beams in a quadratic nonlinear photonic crystals

Ido Dolev[a)] and Ady Arie
Department of Physical Electronics, Fleischman Faculty of Engineering, Tel-Aviv University, Tel-Aviv 69978, Israel

(Received 6 September 2010; accepted 25 September 2010; published online 25 October 2010)

We study three wave mixing processes of accelerating Airy beams in quadratic nonlinear crystals. In order to frequency convert these beams, the nonlinear process should be performed with the Fourier transform of the beam, rather than with the beam itself. This was confirmed experimentally by frequency doubling of one-dimensional and two-dimensional Airy beams in a periodically poled crystal. Sum-frequency or difference frequency process between two accelerating beam enable to set the acceleration rate and direction of the generated beam. © *2010 American Institute of Physics*. [doi:10.1063/1.3504247]

In recent years there has been an increasing interest in Airy beams, i.e., beams whose transverse amplitude dependence at origin is defined by the Airy function. These beams have several unusual features, they are termed "*nondiffracting*" since the infinite Airy wavepacket does not spread as it propagates and "*freely accelerating*," since this wavepacket is centered on a parabolic trajectory in free space.[1–3] These unique features are still observed to a large extent with finite (or truncated) Airy beams, in which the Airy wave function is multiplied by either a Gaussian or an exponential window.[4] The central lobe of these finite beams diffracts much slower as compared to a Gaussian beam with similar transverse dimensions, and retains the acceleration properties. Airy beams were shown to be useful for optical micromanipulation of small particles,[5] for the generation of curved plasma channels in air[6] and for generation of Airy-Bessel light bullets.[7] Moreover, the two-dimensional (2D) Airy beam and the accelerating parabolic beams are the only orthogonal and complete set of nondiffracting and freely accelerating solutions to the 2D paraxial wave equations.[8]

Recently nonlinear optical generation of one-dimensional (1D) Airy beams by a three-wave mixing process was demonstrated,[9–11] where a special 2D asymmetrically poled nonlinear photonic crystal was used to convert an input Gaussian beam into a frequency-converted Airy beam. In this paper we consider a different case, in which all the three interacting waves are Airy beams. We shall study the conditions for efficient conversion of input Airy beams into an output Airy beam at a different frequency. These processes do not require an asymmetrically poled crystal, a standard 1D periodically poled crystal or even a crystal cut for birefringent phase matching can be used. Both 1D Airy beams, as well as 2D beams such as parabolic beams and 2D Airy beams can be generated. This method allows the generation of Airy beams at any wavelength within the transparency range of the crystal (270–4500 nm). An alternative approach of frequency conversion a Gaussian beam and then adding a cubic phase using a linear spatial light modulator (SLM) would have much smaller wavelength coverage, limited by the SLM itself. Moreover, since the damage threshold of nonlinear crystal is much higher than that of SLMs, high intensity Airy beams can be nonlinearly generated. In addition, when a sum-frequency or difference frequency process is performed between two accelerating beams, the acceleration rate and direction of the generated beam is determined by the acceleration coefficient of the two input beams.

Let us consider the nonlinear generation of wave by a three-wave mixing interaction. A straight forward attempt to mix two Airy beams is to simply inject them into a nonlinear crystal that is designed to phase match the wave vectors of the interacting beams, by either birefringent phase matching or quasiphase matching. This is the standard method for doubling plane waves, Gaussian beams or Bessel beams.[12] However, as we will show below, this will not be an optimal choice for Airy beams. For the infinite (nontruncated) 1D Airy beam, the electric field for each one of the two input beams takes the form

$$E_j(x,z) = B_j A_i(x/x_j - a_j z^2)\exp[i\phi_j(x,z)]\exp[ik_{z,j}z] + c.c., \quad (1)$$

where B_j is the beam amplitude, Ai is the Airy function, x_j represents the transverse size of the beam, $k_j(k_{z,j})$ is the wave vector (longitudinal wave vector) and $\phi_j(x,z)$ is the phase. The acceleration rate is $a_j = 1/(4k_j^2 x_j^4)$. In the case of 2D Airy beam, the wave function has a similar shape, but the field is a function of two transverse coordinates. The added contribution to the nonlinearly generated wave from a slab of thickness dz is given by, as follows:

$$\delta E_3(x,z) = i\kappa B_1 B_2 \delta Z Ai(x/x_1 - a_1 z^2) Ai(x/x_2 - a_2 z^2) \times \exp\{i[\phi_1(x,z) \pm \phi_1(x,z) + \Delta k_z z + \phi_{\text{crystal}}(x,z)]\}, \quad (2)$$

where κ is the nonlinear coupling coefficient, $\Delta k_z = k_{z,1} \pm k_{z,2} - k_{z,3}$ is the longitudinal phase mismatch and ϕ_{crystal} is the phase added by the nonlinear crystal. The plus sign is for sum frequency generation (SFG) and the minus sign for difference frequency generation (DFG), whereas in the case of second harmonic generation (SHG) [not shown in Eq. (2)], $k_{z,1} \pm k_{z,2}$ are replaced by either $2k_{z,1}$ or $2k_{z,2}$. For the case of first-order quasiphase matching with a 1D periodic poling with a period Λ we get $\phi_{\text{crystal}} = 2\pi z/\Lambda$, and with properly chosen period it will cancel the longitudinal phase mismatch term. Another possibility is to use birefringent phase matching in order to null the longitudinal phase mis-

a)Author to whom correspondence should be addressed. Electronic mail: idodolev@post.tau.ac.il.

match. Unfortunately, the term δE_3 is proportional to the product of two Airy beams, and in the general case, already at the origin it is not an Airy beam by itself. An exception is when one 1D Airy beam is along the X axis and the other 1D Airy beam is along the Y axis, hence their product will yield a 2D Airy beam. Moreover, the input beams accelerate as they propagate, therefore, hence even if the crystal was set for phase matching a collinear process at the origin, it will not phase the noncollinear process that occurs after the two beams have traveled some distance. The generated wave will not build-up constructively, and it will not follow the spatial shape of the initial beam. As a result, the initial accelerating beams will not generate an accelerating beam at the converted frequency.

Let us consider now a different scheme, frequency mixing of the Fourier transform (FT) of the input beams. As we will show below, this can generate efficiently the FT of the frequency converted Airy beam, which can then be FT using an optical lens. The FT of each one of the Airy beams at origin is given by

$$E_j^f(k) = B_j^f \exp\left(\frac{i x_j^3 k^3}{3}\right) = B_j^f \exp\left(\frac{i k^3}{12 k_j^2 x_j a_j}\right). \qquad (3)$$

Here B_j^f is the amplitude of the beam. These beams have cubic phase dependence, with a coefficient that is inversely proportional to the acceleration. When the two beams are mixed in a nonlinear crystal the added contribution from a slab of length dz is

$$\delta E_3(x,z_0) = i\kappa \cdot B_1^f B_2^f \delta Z \times \exp\left\{ i\left[\frac{x^3}{12 k_1^2 x_1 a_1} \pm \frac{x^3}{12 k_2^2 x_2 a_2} + \Delta k_z z + \phi_{\text{crystal}}(x,z) \right]\right\}. \qquad (4)$$

Here we replaced the spatial frequency k with the transverse coordinate x. Let us assume that the longitudinal phase mismatch is perfectly compensated by the crystal phase term, i.e., $\Delta k_z z + \phi_{\text{crystal}} = 0$. The remaining terms in the exponent are the sum (in the cases of SFG) or difference (in the case of DFG) of two cubic terms. In the case of SHG, one gets twice the cubic phase term of the pump beam. The cubic phase dependence will result in an Airy beam after FT, and its acceleration coefficient will be determined by the sum (or difference) of the two input beams. So far we have considered only one transverse dimension, but the same conclusions apply also to the orthogonal transverse axis. Another option to manipulate the phase is by adding a cubic phase term through the modulation of the nonlinear coefficient, as done on[9] where $\phi_{\text{crystal}}(x,z) = 2\pi z/\Lambda + f_c x^3$. All these cases provide a generated wave with a controlled cubic phase, which can be transformed using a lens to an Airy beam. This is in vast contrast to the direct mixing of the Airy beams, which in general does not yield an Airy beam. It is, therefore, advantageous to mix the FT of the beams, rather than the original Airy beams.

Since the ideal Airy beam carries infinite energy, a more realistic case is that of the truncated Airy beam, in which the wave function is multiplied by a Gaussian or exponential window. The above considerations for three-wave mixing would still be valid, provided that the divergence of the beams owing to their truncation is small throughout interaction length, i.e., the confocal parameter is much larger than the crystal length. Indeed, a numerical simulation of SHG of a 2D Airy beam showed that the cross-correlation of the generated beam with an "ideal" truncated Airy beam is higher than 95%, provided that the ratio of the pump confocal parameter to the crystal length exceeds 25.

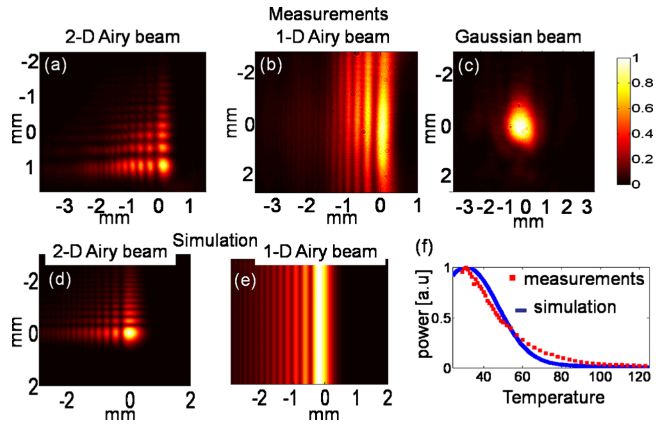

FIG. 1. (Color online) SHG intensity profile of: measured (a) and simulated (d) 2D Airy beam, measured (b) and simulated (e) 1D Airy beam, measured Gaussian beam (c). 1D Airy beam: SH power vs temperature (f).

We have tested experimentally the frequency doubling of an Airy beam. The pump source was a single mode Gaussian beam at 1550 nm, generated by periodically poled stoichiometric lithium tantalate optical parametric oscillator pumped with Nd:YAG laser (1064.5 nm, 10 kHz repetition rate, and 5.5 ns pulse width). The beam radius was expanded to 3 mm and its transverse phase was set using a 512×512 pixels^2 SLM in either 1D or 2D, with total of phase deviation of $-24\ \pi$ to $24\ \pi$. The pump beam with the cubic phase was then imaged using a lens with focal length of 38 mm and magnification of 0.1, producing a Gaussian beam with cubic phase, waist of 300 μm and confocal parameter of 0.67 m, into the center of a 1 cm PPKTP (periodically poled $KTiOPO_4$) with poling period of 24.7 μm. The crystal was placed on an oven at temperature control of 0.1 °C. The output second harmonic (SH) wave was optically FT with another lens to a CCD camera.

In Figs. 1(a)–1(c) we present the experiential results for the SHG of Airy beams, with 2D, 1D and without cubic phase modulation of the pump, respectively. As was also confirmed by split-step Fourier numerical simulations [Figs. 1(d) and 1(e)], SH Airy beams are generated when Gaussian beam with a cubic phase is doubled in the PPKTP crystal. Using pump wave with average powers of 16 mW we measured a 2D SH Airy beam with an average power of 13.1 μW, giving an internal conversion efficiency of 3.7 $\times10^{-6}$ W^{-1}, after taking into account the Fresnel reflections at the uncoated crystal facets. We have also measured the temperature dependence of the process, which is in good agreement to the theoretical predictions, see Fig. 1(f).

We have also examined the case of direct frequency doubling of Airy beams. We used similar setup before the SLM, but this time we performed optical FT of the SLM plane to the center of the crystal using a 2f system. The measured output beam at the crystal output facet [Fig. 2(d)] indeed is not an Airy beam. We have also numerically simulated this process. The evolution of the SH and pump waves inside a 1 cm long PPKTP crystal are shown in Figs. 2(a) and 2(b), respectively. The pump beam inside the crystal is 1D Airy

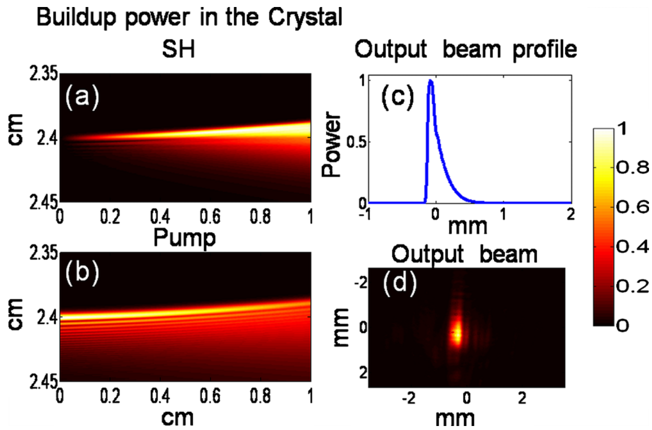

FIG. 2. (Color online) Direct frequency doubling of a fundamental Airy beam. Simulations of the SH buildup power (a), pump power in the crystal (b) and the output beam profile (c). Measured profile of output beam (d).

beam but the SH beam inside the crystal and the output beam profile after the crystal [Fig. 2(c)] do not show the oscillatory behavior of an Airy beam.

We have also used our numerical simulation to analyze SFG and DFG of two truncated 1D Airy beams at 1550 and 1064 nm (Fig. 3). The initial beams were FT, and the nonlinear interaction was performed in a 1 cm long PPKTP crystal with a period of 15.2 μm (35.7 μm), for the SFG (DFG) process. The input beams were assumed to be undepleted and the output beams were FT to obtain 1D Airy beams at 630 nm (SFG) and 3393 nm (DFG). As shown in Eq. (4), the sign of the cubic phase of the generated field can be either positive or negative, hence the generated beam can be accelerated either to the right or to the left. By changing the initial phase ratio between the two generating beams, we can control the acceleration coefficient of the generated beam. This is illustrated in Fig. 4 (Media 1) for DFG. An interesting case occurs when the initial values in Eq. (3) are $x_1 = x_2$ in DFG or $x_1 = -x_2$ in SFG. This eliminates the cubic phase of the output

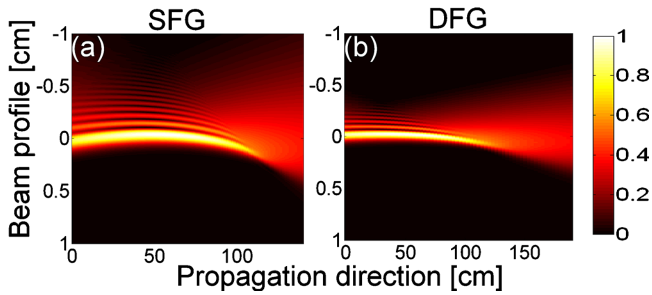

FIG. 3. (Color online) Simulation with input wavelengths of 1550 nm and 1064 nm. SFG: generated Airy beam at 630 nm (a). DFG: generated Airy beam at 3393 nm (b).

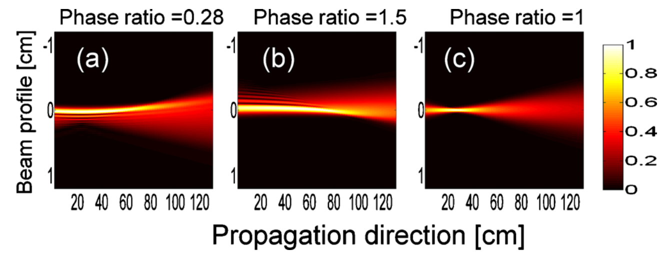

FIG. 4. (Color online) Snapshots from Media 1. DFG of Airy beams with different cubic phase coefficient ratios (x_1/x_2) (enhanced online). [URL: http://dx.doi.org/10.1063/1.3504247.1]

beam, which is not an accelerating Airy beam anymore. It is also interesting to mention that despite the relatively low intensities that SLM can withstand, a high intensity Airy beam can be generated by modulating the phase of a weak beam with the SLM and mixing it with a strong pump in the nonlinear crystal. In addition, DFG process also amplifies the signal beam at 1550 nm. This parametric amplification can be used to overcome the decay of the truncated Airy beams in free-space propagation.

In summary, we have shown by analysis, experiment and numerical simulations that Airy beams can be frequency converted in a quadratic nonlinear crystal, provided that the interaction is performed with the FT of the beams. Moreover, the acceleration magnitude and direction in DFG and SFG interactions are governed by the accelerations of the input beams. Additional flexibility is provided by transversely modulating the nonlinear coefficient of the crystal.[9] In addition to Airy beams, the ability to frequency convert and control the properties of the generated beam may be useful also for accelerating parabolic beams.[8]

This work was supported by COST, action no. MP0702, and the Israel Science Foundation, Grant No. 774/09.

[1]M. V. Berry and N. L. Balazs, Am. J. Phys. **47**, 264 (1979).
[2]G. A. Siviloglou, J. Broky, A. Dogariu, and D. N. Christodoulides, Phys. Rev. Lett. **99**, 213901 (2007).
[3]G. A. Siviloglou, J. Broky, A. Dogariu, and D. N. Christodoulides, Opt. Lett. **33**, 207 (2008).
[4]G. A. Siviloglou and D. N. Christodoulides, Opt. Lett. **32**, 979 (2007).
[5]P. Polynkin, M. Kolesik, J. V. Moloney, G. A. Siviloglou, and D. N. Christodoulides, Science **324**, 229 (2009).
[6]J. Baumgartl, M. Mazilu, and K. Dholakia, Nat. Photonics **2**, 675 (2008).
[7]A. Chong, W. H. Renninger, D. N. Christodoulides, and F. W. Wise, Nat. Photonics **4**, 103 (2010).
[8]M. A. Bandres, Opt. Lett. **33**, 1678 (2008).
[9]T. Ellenbogen, N. Voloch, A. Ganany-Padowicz, and A. Arie, Nat. Photonics **3**, 395 (2009).
[10]I. Dolev, T. Ellenbogen, N. Voloch, and A. Arie, Appl. Phys. Lett. **95**, 201112 (2009).
[11]I. Dolev, T. Ellenbogen, and A. Arie, Opt. Lett. **35**, 1581 (2010).
[12]T. Wulle and S. Herminghaus, Phys. Rev. Lett. **70**, 1401 (1993).