Output power and spectrum of optical parametric generator in the superfluorescent regime

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Received February 26, 2008; revised April 9, 2008; accepted April 14, 2008;
posted April 17, 2008 (Doc. ID 93113); published May 30, 2008

Explicit expressions for the irradiance and signal spectrum of an optical parametric generator were derived. The calculation is in quantitative agreement with measurements of parametric generators with three different lengths of crystals operating in the superfluorescent regime. The measured spectrum was predicted for the entire measured range up to a gain-length product of 16, and the measured signal power was accurately derived up to a gain-length product of 10. © 2008 Optical Society of America

OCIS codes: 190.4223, 190.4410, 190.4970, 190.4975, 270.6630, 230.7405.

An optical parametric generator (OPG) is a nonlinear optical device wherein electromagnetic fields are generated and amplified by parametric interaction with a pump field during a single pass through a nonlinear crystal. The amplified signals start from quantum noise [1,2], which inherently exists in the system. This mechanism is also known as spontaneous parametric downconversion. The quantum noise interacts with the pump wave by second-order nonlinear interaction and generates radiation at different wavelengths.

A quantum-mechanical model of an OPG was first studied by Louisell *et al.* [1], where the formalism of field quantization is used to calculate the expectation values of the number of photons that initiate from the quantum noise in the crystal for a single electromagnetic mode. Since an OPG contains many electromagnetic modes, various theoretical approaches to quantum noise in the optical parametric process were comprehensively discussed [2–8] to appropriately calculate the contribution of the different modes to the total OPG output power. Moreover, a large number of experimental works were done on OPGs in the past 40 years [9–12].

Byer and Harris [2] obtained good agreement between the theoretical and experimental results for the low gain-length regime $g_0L \approx 10^{-3}$, where g_0 is the gain and L is the crystal length. In this case, the dominant process is the spontaneous parametric downconversion process. However, for the high gain-length regime, sometimes called the superfluorescent regime (where $g_0L > 1$), only qualitative comparisons were made between theory and experiment [11]. In this range, the amplification of the spontaneously emitted photons becomes prominent.

The availability of long periodically poled lithium niobate (PPLN) crystals with a high nonlinear coefficient and low loss enables one to reach the superfluorescent regime with long pulse (tens of nanoseconds) high repetition-rate Q-switched lasers. These developments provide an opportunity to re-examine the OPG operation in a quantitative manner. In this Letter, we theoretically and experimentally study the spectral response and the output signal power of an OPG in the high gain-length product regime up to $g_0L \approx 16$. We have found that the theory [8] needs to be refined to match the experimental results. Our theoretical analysis relies on the work of Byer and Harris [2], with the following improvements: (a) an accurate expression for the two-dimensional phase mismatch term [13] was used, (b) a weighting function was added to limit the phase mismatch angle, (c) a nondiffracting Gaussian beam profile was assumed in the transverse direction, and (d) a Gaussian pulse was assumed in the time domain. Using this improved model, we derived a single analytic equation that can be integrated to derive both the OPG output power and the OPG spectrum. We obtained excellent agreement between the measured and calculated spectra for the entire range. In addition, good agreement between the measured and calculated OPG power was obtained for $g_0L \leq 10$.

To find the output energy, the number of generated photons is integrated over all the relevant modes. The number of modes in the frequency range $-\Delta\omega$ to $+\Delta\omega$ and diffraction angles ϕ_x and ϕ_z are calculated depending on the wavenumber k. The total output energy is obtained by summing over the frequencies and angles and over the pump cross section and temporal profile. The pump is assumed to be undepleted in power, and its beam profile, having a waist of w_0, is assumed to be unchanged along the interacting length because the Rayleigh range is much larger than the crystal. In the time domain, a Gaussian pulse of length t_0 is assumed.

The resulting output signal energy of the OPG is

$$E_s = \int_{-\Delta\omega}^{\Delta\omega}\int_{-\varphi_z}^{\varphi_z}\int_{-\varphi_x}^{\varphi_x}\int_{-\text{time}}^{\text{time}}\int_0^r W(\varphi_z,\varphi_x) \times \beta_s \frac{\sinh^2\left(\sqrt{g_0^2 - \frac{\Delta k^2}{4}}L\right)}{\left(g_0^2 - \frac{\Delta k^2}{4}\right)} 2\pi r \mathrm{d}r \mathrm{d}t \mathrm{d}\varphi_x \mathrm{d}\varphi_z \mathrm{d}\omega. \quad (1)$$

The expressions for the gain g_0 and the phase mismatch Δk are

$$g_0^2 = \frac{2\omega_s \omega_i d_{\text{eff}}^2 I_p}{\epsilon_0 n_i(\omega_i) n_s(\omega_s) n_p c^3} \times \exp\left(-2\left(\frac{r}{w_0}\right)^2\right)\exp\left(-2\left(\frac{t}{t_0}\right)^2\right), \quad (2)$$

$$\Delta k = k_p - k_g - k_s \cos\varphi_z \cos\varphi_x - \sqrt{k_i^2 - (k_s \sin\varphi_z)^2 - (k_s \sin\varphi_x \cos\varphi_z)^2}. \quad (3)$$

The phase mismatch term was explicitly calculated in two dimensions (x and z directions) as shown in Fig. 1, where k_p, k_s, k_i, and k_g are the pump, signal, idler, and grating wave vectors, respectively [13]. Unlike previous models [2–8], we did not assume a small angle approximation for the signal diffraction angles φ_z and φ_x. $\beta_s = [h\omega_s^3 n_s^2(\omega_s)/(4\pi^3 c^2)]g_0^2$ is a scaling factor and $W(\phi_z, \phi_x)$ is a weighting function, where a limit on the φ_z, φ_x is set by the pump diffraction angle $\Phi_{\text{diffraction}} = (\lambda / h_s \pi w_0)$

$$W(\varphi_z, \varphi_x) = \exp\left\{-2\left(\frac{\varphi_z^2 + \varphi_x^2}{\phi_{\text{diffraction}}^2}\right)\right\}. \quad (4)$$

In these Eqs. (1)–(4) I_p is the peak pump irradiace, h is the Planck constant, and ε_0 is the dielectric constant in free space; ω and n are the angular frequency and refractive index of the waves, where the subscripts i, s, p refer to the idler, signal, and pump; d_{eff} is the nonlinear coefficient; and c is the speed of light.

The four internal integrals of Eq. (1) provide the spectrum of the OPG, whereas the outer integral provides the total generated energy. Hence, by using only the first four integrals, one obtains the spectrum of the device. For numerically calculating the OPG power, an extended Simpson rule [14] was used for the integration. A five-dimensional array was defined and the numerical integration was calculated on one dimension each time. In the calculation, we assumed $d_{\text{eff}} = 17$ pm/V for both PPLN and periodically poled $MgO:LiNbO_3$ (PPMgLN) crystals, and the refractive indices were calculated using the Sellmeier equation of Dmitriev *et al.* [15].

The experimental setup consists of an $Nd:YVO_4$ Q-switched laser (at a 1.064 μm wavelength) with a 25 ns pulse length and 10 kHz repetition rate. The laser is linearly polarized, and its beam quality is $M^2 < 1.1$. The pump power was controlled with a $\lambda/2$ retardation wave plate and a polarizer. The pump beam was focused into the center of the nonlinear crystal to a beam waist radius of 195 μm. In this experiment we used three different crystals: 35 and 50 mm long PPLN crystals and an 80 mm long PPMgLN crystal [16]. In all three crystals the thickness was 1 mm and the poling period was 28 μm. The phase-matched signal and idler wavelengths were 1.45 and 4 μm, respectively.

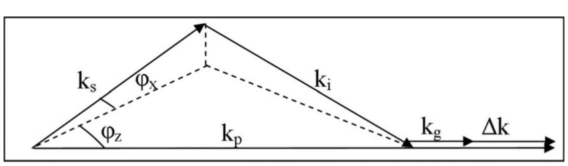

Fig. 1. Phase mismatch diagram.

The crystals facets were polished at 5° and 1° with respect to the domain walls at the entrance and exit facets, respectively, to prevent oscillation of the generated fields within the crystal. The crystals were placed in an oven and heated (to avoid photorefractive damage) to 90 °C in the case of the 80 mm crystal and 180 °C for the other two crystals. To measure the output power of the signal, we inserted calibrated spectral filters into the OPG output beam.

Figure 2 presents the measured and calculated signal output powers obtained with two different PPLN and PPMgLN crystal lengths of 35 and 80 mm, respectively. Each crystal has a different minimum pump power that must be used before the OPG exhibits significant output power. As can be seen, the theoretical model describes this "threshold" effect well. Also shown is the calculation using [8] for the two crystals, which predicts significantly lower power than what was experimentally measured. It should be emphasized that there are no fitting parameters in the theoretical calculation.

Figure 3(a) presents the measured and the calculated signal output powers as a function of the gain-length product up to $g_0 L \approx 10$. The shorter crystal (35 mm) reaches the threshold at lower values of $g_0 L$ and exhibits a large range of agreement with the model, whereas the longer crystal (80 mm) has a higher threshold value and deviates from the model at lower $g_0 L$ values.

Figure 3(b) presents the OPG measured and calculated signal outputs in an 80 mm long PPMgLN crystal at a higher pump power. At a low pump power, there is good agreement between the experimental results and the theoretical model. When the pump power increases, a discrepancy from the theoretical model is revealed. Such a discrepancy also appears in shorter crystals but for a higher pump power. The low generation efficiency regime of the shorter crystals extends to the higher pump power. We empirically estimate that the model prediction is accurate

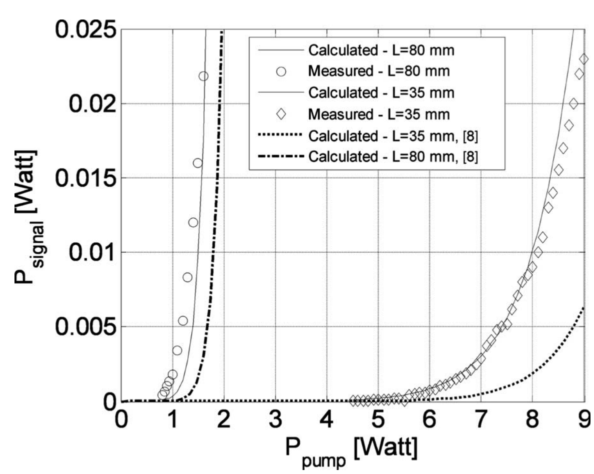

Fig. 2. Measured and calculated average OPG output powers for 35 mm long PPLN and 80 mm long PPMgLN crystals at different pump power levels.

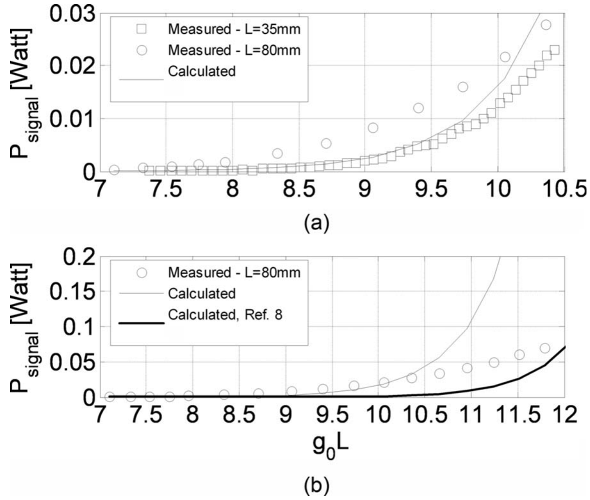

Fig. 3. (a) Measured and calculated average OPG output powers versus gain-length products. (b) Shows larger gain-length products.

up to $g_0L \approx 10$. As seen in Fig. 3(b), the calculated power based on [8] predicts considerably lower values with respect to the experimental results also at the higher gain-length limit.

Figure 4 shows the measured and calculated spectra for three different crystal lengths at different pump powers. There is good agreement between the experimental results and the theoretical model. The spectral width obtained in the 35 mm long crystal was 2.05 nm, whereas the spectral widths obtained in the 50 and 80 mm long crystals were 1.6 and 1.02 nm, respectively. As expected, the spectral width in the long crystal is narrower than in the short crystal, since the acceptance bandwidth of the longer crystal is more narrow [17]. Since we found an inaccuracy in the existing theory [17] we chose to use our model to calculate the spectral bandwidth. Note that the measured bandwidths are comparable to those observed in optical parametric oscillators. The threshold behaviors with respect to the pump power as well as the narrow spectral bandwidths are both characteristic phenomena of the superfluorescent regime.

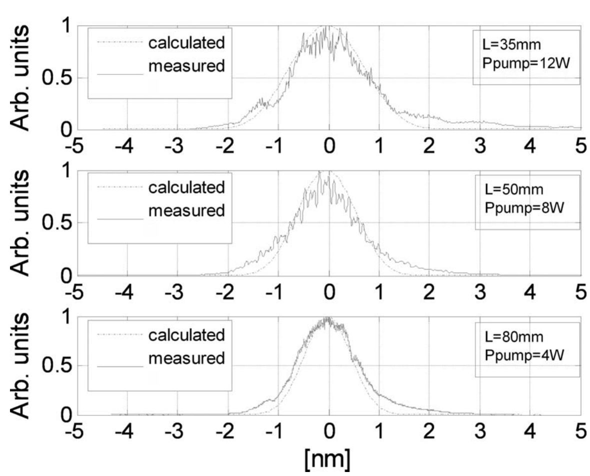

Fig. 4. Measured and calculated spectra of the 1.45 μm signal wave for different pump power levels and crystal lengths.

The measured spectrum agrees well with the theory for the entire measured range up to $g_0L \approx 16$, whereas the measured power is in agreement with the theory up to $g_0L \approx 10$. We believe that this is caused by the method of integration, whereby each integral was calculated on one dimension each time. Since the spectrum requires only four-dimensional integration, it is less vulnerable to integration errors with respect to the five-dimensional integration for the power calculation.

In summary, we introduced a simple analytical expression to calculate the signal output power and the signal spectrum of an OPG, which is also valid in the superfluorescent regime. We have improved the existing model by explicitly considering a Gaussian pump beam having Gaussian time dependence (Gaussian pulse) by adding a weighting function to limit the phase-mismatch angle and by considering an explicit expression for the phase mismatch. The power model predictions were found to be accurate for the three different crystal lengths and for gain-length products up to $g_0L \approx 10$. Furthermore, excellent agreement was found between the measured and the calculated spectra over the entire measured range up to $g_0L \approx 16$.

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